Worksheet #4

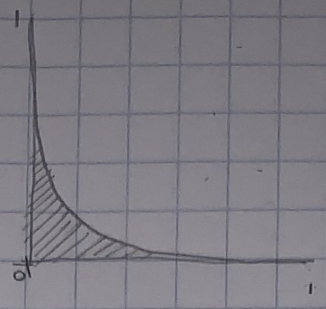
① $x^{2/3}+y^{2/3}=1$

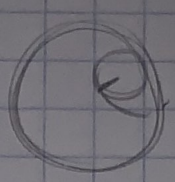
$\int_0^1 \pi x^2\,dy$

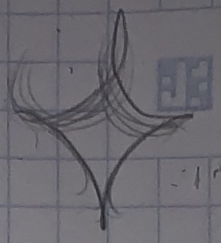
$(x^2=(1-y^{2/3})^3)$

$\pi\int_0^1 1-3y^{2/3}+3y^{4/3}-y^2\,dy$

$=\frac{16}{105}\pi$







② Compute the volume of the solid rotated around x-axis using cylindrical shells.

$\int_0^1 2\pi xy\,dy$

$x=(1-y^{2/3})^{3/2}$

$=\int_0^1 2\pi y(1-y^{2/3})^{3/2}\,dy$

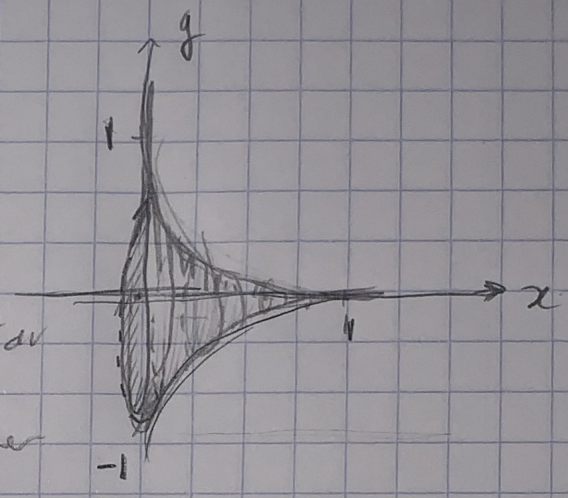
$u^3=y^2 \quad 3u^2\,du=2y\,dy$

$\int_0^1 3\pi u^2(1-u)^{3/2}\,du$

$v=1-u,\ dv=-du$

$-3\pi\int_0^1(1-v)^2v^{3/2}\,dv=3\pi\int_0^1 v^{3/2}-2v^{5/2}+v^{7/2}\,dv$

$=\frac{16}{105}\pi$

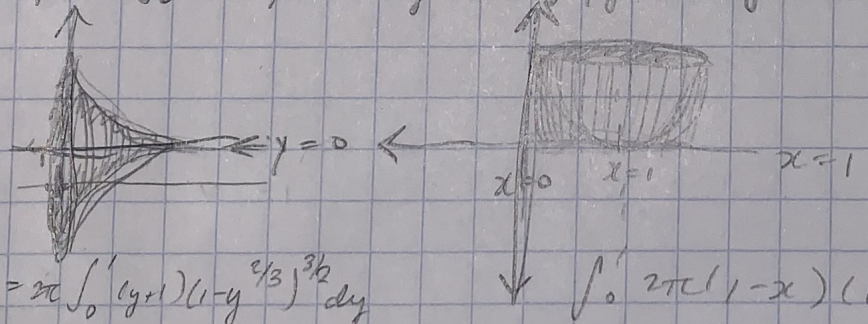


Volumes are the same because rotation around either axis gives the same volume.

③ Set up integrals for rotating around $x=1$ & $y=-1$

$x=1$: $\int_0^1 2\pi(1-x)(1-x^{2/3})^{3/2}\,dx$

$y=-1$: $\int_0^1 2\pi(y+1)x\,dy=2\pi\int_0^1(y+1)(1-y^{2/3})^{3/2}\,dy$



$=2\pi\int_0^1(y+1)(1-y^{2/3})^{3/2}\,dy$

$\int_0^1 2\pi(1-x)(1-x^{2/3})^{3/2}\,dx$